STAT 400
Spring 2020

# Discussion 03

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## 1. 1.2-4 1.3-6 1.3-6

The "eating club" is hosting a make-your-own sundae party at which the following are provided:

| **Ice Cream Flavors** | **Toppings** |
|---|---|
| Chocolate | Caramel |
| Cookies 'n' cream | Hot fudge |
| Strawberry | Marshmallow |
| Vanilla | M&M's |
| | Nuts |
| | Strawberries |

a) How many sundaes are possible using one flavor of ice cream and three different toppings?

b) How many sundaes are possible using one flavor of ice cream and from zero to six toppings?

c) How many different combinations of flavors of three scoops of ice cream are possible if it is permissible to make all three scoops the same flavor?

## 2.

A laboratory has 15 rats, 7 white, 6 gray and 2 brown. Suppose 5 rats will be selected at random and assigned to an experimental drug.

a) Find the probability that 2 white, 2 gray and 1 brown rats were selected.

b) Find the probability that 3 white and 2 gray rats were selected.

c) Find the probability that all five rats selected have the same color.

d) Find the probability that all five rats selected have different color.

**3.** A new flavor of toothpaste has been developed. It was tested by a group of 15 people. Nine of the group said they liked the new flavor, and the remaining 6 indicated they did not. Five of the 15 are selected at random to participate in an in-depth interview.

a) What is the probability that of those selected for the in-depth interview 4 liked the new flavor and 1 did not?

b) What is the probability that of those selected for the in-depth interview at least 4 liked the new flavor?

**4.** **2.1-10** **2.1-10** 2.1-10

Suppose there are 3 defective items in a lot (collection) of 50 items. A sample of size 10 is taken at random and without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains

a) Exactly one defective item.

b) At most one defective item.

**5 – 6.** Sally sells seashells by the seashore. The daily sales X of the seashells have the following probability distribution:

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 1 | 0.25 |
| 2 | 0.20 |
| 3 | 0.15 |
| 4 | 0.10 |

**5.** a) Find the missing probability $f(0) = P(X = 0)$.

b) Find the probability that she sells at least three seashells on a given day.

c) Find the expected daily sales of seashells.

d) Find the standard deviation of daily sales of seashells.

**6.** Suppose each shell sells for \$5.00. However, Sally must pay \$3.00 daily for the permit to sell shells. Let $Y$ denote Sally's daily profit. Then $\mathbf{Y = 5 \cdot X - 3}$.

e) Find the probability that Sally's daily profit will be at least \$10.00.

f) Find Sally's expected daily profit.

g) Find the standard deviation of Sally's daily profit.

**7.** How much wood would a woodchuck chuck if a woodchuck could chuck wood? Let $X$ denote the amount of wood a woodchuck would chuck per day (in cubic meters) if a woodchuck could chuck wood. Find the missing probability $f(0) = P(X = 0)$, the average amount of wood a woodchuck would chuck per day, $E(X)$, and the variance $\mathrm{Var}(X)$.

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 5 | 0.30 |
| 10 | 0.40 |
| 15 | 0.10 |

**8.** *Initech* does part of its business via Internet. However, because of the high traffic, the server that carries the company's website often crashes. The probability distribution of the number of server crashes per month, X, is given below:

| $x$ | $f(x)$ |
|---|---|
| 0 | 0.15 |
| 1 | 0.35 |
| 2 | 0.45 |
| 3 | 0.05 |

If the server does not crash, *Initech*'s monthly profit from the website is \$80,000. However, each time the server crashes, *Initech* loses \$30,000. Therefore, *Initech*'s monthly profit from the website (in thousands of \$) is $Y = 80 - 30 \cdot X$. Find *Initech*'s average monthly profit from the website and its standard deviation.

**9.** Consider $f(x) = c\left(\frac{1}{4}\right)^x$, $x = 1, 2, 3, 4, \ldots$.

a) Find $c$ such that $f(x)$ satisfies the conditions of being a p.m.f. for a random variable X.

b) Find the expected value of X.

c) Find $P(\text{X is odd})$.

**10.** Find $E(X)$ for the following discrete probability distributions:

a) $f(0) = \frac{7}{8}$, $f(k) = \frac{1}{3^k}$, $k = 2, 4, 6, 8, \ldots$.

(possible values of X are even non-negative integers: 0, 2, 4, 6, 8, …).

b) $f(1) = \ln 3 - 1$, $f(k) = \frac{(\ln 3)^k}{k!}$, $k = 2, 3, 4, \ldots$.

(possible values of X are positive integers: 1, 2, 3, 4, …).

**11.** Each of three balls is randomly placed into one of three bowls. Let X denote the number of empty bowls.

a) Find the probability distribution of X.

"Hint": $f(0) = P(X = 0)$ and $f(2) = P(X = 2)$ are easier to find.

b) Find $E(X)$.

**12.** Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f.

$$f(x) = \frac{5-x}{10}, \qquad x = 1,2,3,4.$$

a) Find $E(X)$, the expected number of days the patient needs to be in the hospital.

b) **2.2-3** **2.2-5** 2.2-5

If the patient is to receive $200 from the insurance company for each of the first two days in the hospital and $100 for each day after the first two days, what is the expected payment for the hospitalization?

"Hint": If the patient spends three days in the hospital, the payment is $500.