

1. **2.3-11**    **2.5-3**    2.5-3

If the moment-generating function of  $X$  is

$$M_X(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

Find the mean, variance, and pmf of  $X$ .

2. Suppose a discrete random variable  $X$  has the following probability distribution:

$$f(0) = \frac{7}{8}, \quad f(k) = \frac{1}{3^k}, \quad k = 2, 4, 6, 8, \dots$$

(possible values of  $X$  are even non-negative integers:  $0, 2, 4, 6, 8, \dots$ ).

Recall Week 02 Discussion Problem 1 (a):      this is a valid probability distribution.

- a) Find the moment-generating function of  $X$ ,  $M_X(t)$ . For which values of  $t$  does it exist?
- b) Use  $M_X(t)$  to find  $E(X)$ .

3. Suppose a discrete random variable  $X$  has the following probability distribution:

$$f(1) = \ln 3 - 1, \quad f(k) = \frac{(\ln 3)^k}{k!}, \quad k = 2, 3, 4, \dots$$

(possible values of  $X$  are positive integers:  $1, 2, 3, 4, \dots$ ).

Recall Week 02 Discussion Problem 1 (b):      this is a valid probability distribution.

- a) Find the moment-generating function of  $X$ ,  $M_X(t)$ . For which values of  $t$  does it exist?
- b) Use  $M_X(t)$  to find  $E(X)$ .

4. Suppose the moment-generating function of  $X$  is

$$M_X(t) = 0.1 e^{2t} + 0.3 e^{4t} + 0.6 e^{7t}.$$

- a) Find  $\mu = E(X)$ .                                      b) Find  $\sigma = SD(X)$ .

5. Suppose a discrete random variable  $X$  has the following probability distribution:

$$f(k) = P(X = k) = a^k, \quad k = 2, 3, 4, 5, 6, \dots, \quad \text{zero otherwise.}$$

- a) Find the value of  $a$  that makes this is a valid probability distribution.  
b) Find  $P(X \text{ is even})$ .  
c) Find the moment-generating function of  $X$ ,  $M_X(t)$ . For which values of  $t$  does it exist?  
d) Find  $E(X)$ .

6. Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \frac{C}{x^4}, \quad x > 5, \quad \text{zero otherwise.}$$

- a) Find the value of  $C$  that would make  $f(x)$  a valid probability density function.  
b) Find the cumulative distribution function of  $X$ ,  $F(x) = P(X \leq x)$ .  
“Hint”: Should be  $F(5) = 0$ ,  $F(\infty) = 1$ .  
c) Find the probability  $P(6 < X < 10)$ .  
f) Find the 80th percentile of the distribution of  $X$ ,  $\pi_{0.80}$ .  
g) Find the expected value of  $X$ ,  $E(X)$ .  
h) Find the standard deviation of  $X$ ,  $SD(X)$ .

7. Let  $X$  be a continuous random variable with the probability density function

$$f(x) = Cx^2, \quad 3 \leq x \leq 9, \quad \text{zero otherwise.}$$

- a) Find the value of  $C$  that would make  $f(x)$  a valid probability density function.
- b) Find the probability  $P(X < 5)$ .
- c) Find the probability  $P(X > 7)$ .
- d) Find the mean of the probability distribution of  $X$ .
- e) Find the median of the probability distribution of  $X$ .

8. Suppose a random variable  $X$  has the following probability density function:

$$f(x) = \cos x, \quad 0 < x < \frac{\pi}{2}, \quad \text{zero otherwise.}$$

- a) Find  $P(X < \frac{\pi}{4})$ .
- b) Find  $\mu = E(X)$ .
- c) Find the median of the probability distribution of  $X$ .

9. Let  $X$  be a continuous random variable with the probability density function

$$f(x) = 6x(1-x), \quad 0 < x < 1, \quad \text{zero elsewhere.}$$

Compute  $P(\mu - 2\sigma < X < \mu + 2\sigma)$ .

10. Suppose a random variable  $X$  has the following probability density function:

$$f(x) = xe^x, \quad 0 < x < 1, \quad \text{zero otherwise.}$$

- a) Find  $P(X < \frac{1}{2})$ .
- b) Find  $\mu = E(X)$ .
- c) Find the moment-generating function of  $X$ ,  $M_X(t)$ .

**11.** Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \begin{cases} c|x-3|, & 0 < x < 8, \\ 0, & \text{otherwise.} \end{cases}$$

- a) Find the value of  $c$  that makes  $f(x)$  a valid probability density function.
- b) Find the probability  $P(X < 5)$ .
- c) Find the median of the probability distribution of  $X$ .
- d) Find the mean of the probability distribution of  $X$ .
- e) Find the variance of the probability distribution of  $X$ .