

1. Let X and Y have the joint probability density function

$$f_{X,Y}(x,y) = Cxy^2, \quad 0 < x < y < 1, \quad \text{zero otherwise.}$$

- a) What must the value of C be so that $f_{X,Y}(x,y)$ is a valid joint p.d.f.?
- b) Find the marginal probability density function of X , $f_X(x)$. *Include its support.*
- c) Find the marginal probability density function of Y , $f_Y(y)$. *Include its support.*
- d) Find $P(X+Y < 1)$.
- e) Let $a > 1$. Find $P(Y < aX)$.
- f) Are X and Y independent? If not, find $\text{Cov}(X, Y)$.

2. Let S and T have the joint probability density function

$$f_{S,T}(s,t) = \frac{1}{t}, \quad 0 < s < 1, \quad s^2 < t < s.$$

- a) Find $f_S(s)$ and $f_T(t)$.
- b) Find $E(S)$ and $E(T)$.
- c) Find the correlation coefficient ρ_{ST} .

3. Let X and Y be random variables with

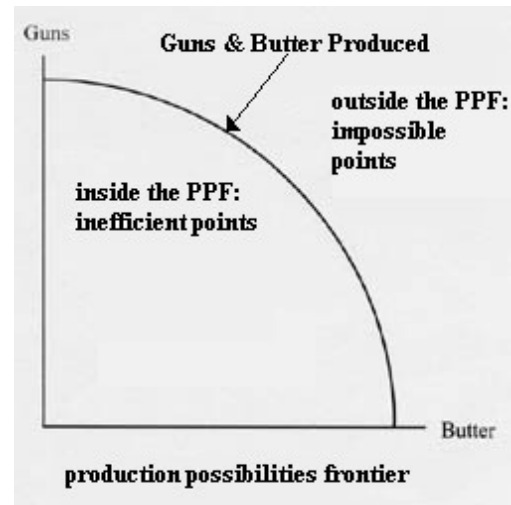
$$E(X) = \mu_X = 25, \quad \text{SD}(X) = \sigma_X = 4,$$

$$E(Y) = \mu_Y = 40, \quad \text{SD}(Y) = \sigma_Y = 3, \quad \text{Corr}(X, Y) = \rho = -0.50.$$

- a) Find $E(2X + 5Y)$ and $\text{SD}(2X + 5Y)$.
- b) Find $E(4Y - 5X)$ and $\text{SD}(4Y - 5X)$.

7. Every month, the government of Neverland spends X million dollars purchasing guns and Y million dollars purchasing butter. Assume X and Y are independent, X has a Normal distribution with mean 265 and standard deviation 40 (in millions of dollars), and Y has a Normal distribution with mean 170 and standard deviation 30 (in millions of dollars).
- Find the probability that the government of Neverland spends more on guns than on butter during a given month. That is, find $P(X > Y)$.
 - Find the probability that the government of Neverland spends more on guns than twice the amount it spends on butter during a given month. That is, find $P(X > 2Y)$.
 - Find the probability that the government of Neverland exceeds the 500-million spending limit during a given month. That is, find $P(X + Y > 500)$.

8. The previous problem is not very realistic – X and Y should NOT be independent, but the correlation coefficient of X and Y should be negative. Assume X has a Normal distribution with mean 265 and standard deviation 40 (in millions of dollars), and Y has a Normal distribution with mean 170 and standard deviation 30 (in millions of dollars). Assume also that the correlation coefficient of X and Y is $\rho = -0.56$. Assume that any linear combination of X and Y is normally distributed (that would be the case if X and Y jointly have a Bivariate Normal distribution [4.5 4.4 5.6]).



- Find the probability that the government of Neverland spends more on guns than on butter during a given month. That is, find $P(X > Y)$.
- Find the probability that the government of Neverland exceeds the 500-million spending limit during a given month. That is, find $P(X + Y > 500)$.

“Hint”: In each case, find the mean and the variance of the appropriate linear combination of X and Y first.