

1. Consider a 6-pack of soda. Suppose that the amount of soda in each can follows a normal distribution with mean 12.06 oz and standard deviation 0.15 oz. Assume that all cans are filled independently of each other. Find the probability of the following:
  - a) a can is underfilled, i.e. there is less than 12 oz of soda in a can;
  - b) all 6 cans are underfilled;
  - c) at least one of the 6 cans is underfilled;
  - d) exactly 2 of the 6 cans are underfilled;
  - e) the average amount of soda in these 6 cans is less than 12 oz.
  
2. The weights of the eggs at a particular farm are normally distributed with the mean weight of 1.75 oz and standard deviation 0.25 oz.
  - a) Find the probability that a randomly selected egg weighs over 1.5 oz.
  - b) If three dozen eggs (36 eggs) are randomly and independently selected, what is the probability that exactly 30 of them weigh over 1.5 oz?
  - c) What is the probability that the total weight of (randomly and independently selected) three dozen eggs (36 eggs) is over 60 oz?
  
3. **5.6-8**      **5.6-8**      ~~5.4-8~~  
Let  $X$  equal the weight in grams of a miniature candy bar. Assume that  $\mu = E(X) = 24.43$  and  $\sigma^2 = \text{Var}(X) = 2.20$ . Let  $\bar{X}$  be the sample mean of a random sample of  $n = 30$  candy bars. Find:
  - (a)  $E(\bar{X})$ .
  - (b)  $\text{Var}(\bar{X})$ .
  - (c)  $P(24.17 < \bar{X} < 24.82)$ , approximately.

4. **5.6-4**      **5.6-4**      ~~5.4-4~~

Approximate  $P(39.75 < \bar{X} < 41.25)$ , where  $\bar{X}$  is the mean of a random sample of size  $n = 32$  from a distribution with mean  $\mu = 40$  and variance  $\sigma^2 = 8$ .

5. The weight of an almond varies with mean 0.047 ounce and standard deviation 0.004 ounce.
- a) What is the probability (approximately) that the total weight (of a random sample) of 64 almonds is greater than 3 ounces?
- b) Determine the sample size (the number of almonds) needed to have the probability of at least 0.90 that the total weight is greater than 16 ounces.
6. An instructor gives a test to a class containing several hundred students. It is known that the standard deviation of the scores is 14 points. A random sample of 49 scores is obtained.
- a) What is the probability that the average score of the students in the sample will differ from the overall average by more than 2 points?
- b) What is the probability that the average score of the students in the sample will be within 3 points of the overall average?
7. Let  $X_1$  and  $X_2$  be independent with normal distributions  $N(6, 1)$  and  $N(7, 1)$ , respectively. Find  $P(X_1 > X_2)$ .
- Hint: Write  $P(X_1 > X_2) = P(X_1 - X_2 > 0)$  and determine the distribution of  $X_1 - X_2$ .
8. Compute  $P(X_1 + 2X_2 - 2X_3 > 7)$ , if  $X_1, X_2, X_3$  are i.i.d. with common distribution  $N(1, 4)$ .

9. Let  $X_1, X_2, \dots, X_{70}$  be a random sample of size  $n = 70$  from a distribution with p.d.f.
- $$f(x) = \frac{1}{72} (6 - x)^2, \quad 0 < x < 6, \quad \text{zero elsewhere.}$$
- Find  $P(\bar{X} < 1.6)$  approximately.
10. A machine operation produces widgets whose diameters are normally distributed, with a mean of 4.97 inches and a standard deviation of 0.06 inches. Suppose that specifications require that the widget diameter be 5.00 inches plus or minus 0.12 inches (that is, between 4.88 and 5.12 inches).
- What proportion of the production will be unacceptable?
  - Suppose 25 widgets are independently and randomly selected from the production process. What is the probability that exactly 2 of the 25 will be unacceptable?
  - A quality control inspector selects 25 widgets from the production independently and at random. If the average diameter of the selected widgets is within 0.06 inches of 5.00 inches (that is, between 4.94 and 5.06 inches), the production process is allowed to continue. However, if the average diameter of the selected widgets is not within 0.06 inches of 5.00 inches, the production process is stopped and the machine is checked. What is the probability that the production process will be stopped after examining a random sample of 25 widgets?
11. 10. (continued)
- After a disgruntled employee kicked the machine, the mean shifted to 4.91 inches, and the standard deviation shifted to 0.10 inches, while the distribution of the diameters remained normal.
- What proportion of the production will be unacceptable after the machine is kicked?
  - After the machine was kicked, 25 widgets are independently and randomly selected from the production process. What is the probability that at least 7 of the 25 will be unacceptable?
  - After the machine was kicked, what is the probability that the production process will be stopped after examining a random sample of 25 widgets?

12. 6.4-1 6.4-1 6.2-1

Let  $X_1, X_2, \dots, X_n$  be a random sample from:

$N(\mu, \sigma^2)$        $\mu$  unknown,       $\sigma$  known.

Show that  $\hat{\mu} = \bar{X}$  is the MLE for  $\mu$ .

13. 6.4-2 6.4-2 6.2-2

Let  $X_1, X_2, \dots, X_n$  be a random sample from:

$N(\mu, \sigma^2)$        $\mu$  known,       $\sigma$  unknown.

Show that  $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \mu)^2$  is the MLE for  $\sigma^2$ .

14. Let  $\theta > 0$  and let  $X_1, X_2, \dots, X_n$  be a random sample from the distribution with the probability density function

$$f_X(x) = f_X(x; \theta) = \frac{\theta}{2\sqrt{x}} e^{-\theta\sqrt{x}}, \quad x > 0.$$

a) Find the method of moments estimator  $\tilde{\theta}$  of  $\theta$ .

b) Suppose  $n = 4$ , and  $x_1 = 0.01$ ,  $x_2 = 0.04$ ,  $x_3 = 0.09$ ,  $x_4 = 0.36$ .

Find the method of moments estimate  $\tilde{\theta}$  of  $\theta$ .

c) Find the maximum likelihood estimator  $\hat{\theta}$  of  $\theta$ .

d) Suppose  $n = 4$ , and  $x_1 = 0.01$ ,  $x_2 = 0.04$ ,  $x_3 = 0.09$ ,  $x_4 = 0.36$ .

Find the maximum likelihood estimate  $\hat{\theta}$  of  $\theta$ .

e)\* Find a closed-form expression for  $E(X^k)$ ,  $k > -\frac{1}{2}$ .      “Hint”: Consider  $u = \theta\sqrt{x}$ .