STAT 400
Spring 2020

Discussion 11 Answers

Stepanov
Dalpiaz
Nguyen

1. A random sample of size $n = 16$ from $N(\mu, \sigma^2 = 64)$ yielded $\overline{x} = 85$.

Construct the following confidence intervals for μ:

$\overline{x} = 8$ $\sigma = 8$ $n = 16$

σ is known. The confidence interval : $\overline{X} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$.

a) 95%.

$\alpha = 0.05$ $\alpha/2 = 0.025.$ $z_{\alpha/2} = 1.96.$

$85 \pm 1.96 \cdot \dfrac{8}{\sqrt{16}}$ **85 ± 3.92** **(81.08 ; 88.92)**

b) 90%.

$\alpha = 0.10$ $\alpha/2 = 0.05.$ $z_{\alpha/2} = 1.645.$

$85 \pm 1.645 \cdot \dfrac{8}{\sqrt{16}}$ **85 ± 3.29** **(81.71 ; 88.29)**

c) 80%.

$\alpha = 0.20$ $\alpha/2 = 0.10.$ $z_{\alpha/2} = 1.28.$

$85 \pm 1.28 \cdot \dfrac{8}{\sqrt{16}}$ **85 ± 2.56** **(82.44 ; 87.56)**

OR

$\alpha = 0.20$ $\alpha/2 = 0.10.$ $z_{\alpha/2} = 1.282.$

$85 \pm 1.282 \cdot \dfrac{8}{\sqrt{16}}$ **85 ± 2.564** **(82.436 ; 87.564)**

2. What is the minimum sample size required for estimating μ for $N(\mu, \sigma^2 = 64)$ to within ± 3 with confidence level

$\varepsilon = 10$, $\sigma = 8$.

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{z_{\alpha/2} \cdot 8}{3} \right)^2 .$$

a) 95%. $\alpha = 0.05$ $\alpha/2 = 0.025.$ $z_{\alpha/2} = 1.96.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.96 \cdot 8}{3} \right)^2 \approx 27.318.$$ Round **up**. $n =$ **28**.

b) 90%. $\alpha = 0.10$ $\alpha/2 = 0.05.$ $z_{\alpha/2} = 1.645.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.645 \cdot 8}{3} \right)^2 \approx 19.243.$$ Round **up**. $n =$ **20**.

c) 80%. $\alpha = 0.20$ $\alpha/2 = 0.10.$ $z_{\alpha/2} = 1.28.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.28 \cdot 8}{3} \right)^2 \approx 11.651.$$ Round **up**. $n =$ **12**.

OR

$\alpha = 0.20$ $\alpha/2 = 0.10.$ $z_{\alpha/2} = 1.282.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.282 \cdot 8}{3} \right)^2 \approx 11.687.$$ Round **up**. $n =$ **12**.

3. Suppose the overall (population) standard deviation of the bill amounts at a supermarket is $\sigma = \$13.75$.

a) Find the probability that the sample mean bill amount will be within \$2.00 of the overall mean bill amount for a random sample of 121 customers.

Need $P(\mu - 2.00 \le \overline{X} \le \mu + 2.00) = ?$

$n = 121$ – large

Central Limit Theorem: $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} = Z.$

$$P(\mu - 2.00 \le \overline{X} \le \mu + 2.00) = P\left(\frac{(\mu - 2.00) - \mu}{13.75 / \sqrt{121}} \le Z \le \frac{(\mu + 2.00) - \mu}{13.75 / \sqrt{121}} \right)$$

$$= P(-1.60 \le Z \le 1.60) = 0.9452 - 0.0548 = \mathbf{0.8904}.$$

b) What is the minimum sample size required for estimating the overall mean bill amount to within \$2.00 with 95% confidence?

$\varepsilon = 2.00,\quad \sigma = 13.75,\quad \alpha = 0.05,\quad \alpha/2 = 0.025,\quad z_{\alpha/2} = z_{0.035} = 1.96.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.96 \cdot 13.75}{2.00} \right)^2 = 181.575625.$$

Round **up**. $n = \mathbf{182}$.

4. 3. (continued)

The supermarket selected a random sample of 121 customers, which showed the sample mean bill amount of \$78.80.

$\overline{X}$ = \$78.80, σ = \$13.75, n = 121.

c) Construct a 95% confidence interval for the overall mean bill amount at this supermarket.

σ is known. $n = 121$ – large.

The confidence interval for μ: $\overline{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

$\alpha = 0.05$. $\alpha/2 = 0.025$. $z_{\alpha/2} = z_{0.025} = 1.96$.

$78.80 \pm 1.96 \cdot \frac{13.75}{\sqrt{121}}$ **78.80 ± 2.45** **(76.35 ; 81.25)**

d) Suppose the supermarket puts Alex in charge of computing the confidence interval, and he gets the answer (76.15 , 81.45). Alex says that he used a different confidence level, but other than that did everything correctly. Find the confidence level used by Alex.

$\overline{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $81.45 - 78.80 = 78.80 - 76.15 = 2.65$.

$2.65 = z_{\alpha/2} \cdot \frac{13.75}{\sqrt{121}}$ $z_{\alpha/2} = 2.12$.

$\alpha/2$ = Area to the right of 2.12 = 0.0170. $\alpha = 2 \cdot 0.0170 = 0.0340$.

Confidence level = $100 \cdot (1 - \alpha)\%$ = **96.6%**.