1.2 - Methods of Enumeration (Counting)

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Multiplication Principle (Fundamental Rule of Counting):

If there are n events and event i can occur in N_i possible ways, then the number of ways in which the sequence of n events may occur is

$$N_1 \cdot N_2 \cdot ... \cdot N_n$$

Example 1:
Manager of a radio station decided that every day the broadcast will start with one of the 9 Beethoven Symphonies, followed by one of Mozart's 27 Piano Concertos, followed by one of Schubert's 15 String Quartets. Approximately how many years can the station do that without repeating the program?

Example 2:
In how many orders can the names of 5 candidates for the same office be listed on a ballot?

$$n! = 1 \cdot 2 \cdot ... \cdot (n-1) \cdot n \qquad n! = n \cdot (n-1)!$$
$$n! = n \cdot (n-1) \cdot ... \cdot 2 \cdot 1 \qquad 0! = 1$$

Example 3:
How many ways are there of scrambling the letters of the word SCRAMBLE ?

Example 4:
Eight horses are entered in a race in which bets are placed on which horse will win, place, and show (that is, finish first, second and third). Suppose that the race is run and there are no ties. In how many ways can the win, place, and show be taken?

Permutations are the possible ordered selections of r objects out of a total of n objects. The number of permutations of n objects taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

Example 5:
At *Momma Leona's Pizza* you can get a pizza with or without each of eight different toppings. How many different three-topping pizzas can you get at *Momma Leona's Pizza* if each topping can be put on a pizza at most once?
(The order in which the toppings toppings are selected does not matter.)

Combinations are the possible selections of r items from a group of n items *regardless of the order of selection*. The number of combinations of n objects taken r at a time is

$$_nC_r = \binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$$

	order of the selection is important	order of the selection is **not** important
repetitions allowed (w/ replacement)	n^r	$n+r-1\ \mathrm{C}\ r$
repetitions **not** allowed (w/o replacement)	$n\ \mathrm{P}\ r = \frac{n!}{(n-r)!}$	$n\ \mathrm{C}\ r = \frac{n!}{r!\cdot(n-r)!}$

Example 6:
The Baskin-Robbins Ice Cream Stores have 31 flavors of ice cream.

a) How many different 3-scoop ice cream cones are possible if you are allowed to repeat flavors and want the scoops put on the cone in a particular order?

b) How many different 3-scoop ice cream cones are possible if each scoop is a different flavor and you want the scoops put on the cone in a particular order?

c) How many different 3-scoop cones are possible if each scoop is a different flavor and you don't care about their order on the cone?

d) How many different 3-scoop ice cream cones are possible if you are allowed to repeat flavors, but the order in which the scoops are placed into the cone is not important?

Example 7:
To play Michigan Lotto, a person must pick 6 numbers from 49 numbers.

a) If the player matches all 6 numbers (6 of 6) drawn, he/she wins the grand prize jackpot. Find the probability of winning the jackpot.

b) Find the probability of guessing correctly 4 out of 6 numbers.

Example 8:
How many ways are there of scrambling the letters of the word BANANA ?

Example 9:
A student bookstore has 18 STAT 400 textbooks: 9 are new, 6 are used, and 3 are abused.

a) How many ways are there to arrange the textbooks on the shelf?

b) Eight students come to the store to buy a STAT 400 textbook. Suppose that the purchased textbooks are selected at random. What is the probability that 4 of the students would get a new book, 3 would get a used one, and 1 student would get an abused textbook?