STAT 400
UIUC

1.4 – Independent Events

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Events **A and B** are **independent** if and only if

$$\mathbf{P(B \mid A) = P(B) \qquad P(A \mid B) = P(A)}$$

$$\mathbf{P(A \cap B) = P(A) \cdot P(B)}$$

Example 1:
The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10. Are events {a student owns a bicycle} and {a student owns a car} independent?

Events **A, B and C** are **independent** if and only if

$$P(A \cap B) = P(A) \cdot P(B),$$

and $P(A \cap C) = P(A) \cdot P(C)$,

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Example 2:
A girl is told by her boyfriend that she is "one in a billion." She has a dimple in her chin, probability $1/100$, eyes of different colors, probability $1/1{,}000$, and is absolutely crazy about mathematics, probability $1/10{,}000$.

a) Do these events seem to be independent or dependent?

b) Show why the girl is "one in a billion."

Example 3:

Bart and Nelson talked Milhouse into throwing water balloons at Principal Skinner. Suppose that Bart hits his target with probability 0.80, Nelson misses 25% of the time, and Milhouse hits the target half the time. Assume that their attempts are independent of each other.

a) Find the probability that all of them will hit Principal Skinner.

b) Find the probability that exactly one of the boys will hit Principal Skinner.

c) Find the probability that at least one of the boys will hit Principal Skinner.

Idea: P(at least one of A_i occurs) = 1 – P(none of A_i occurs)

$$\mathbf{P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n) = 1 - P((\text{not } A_1) \text{ and } (\text{not } A_2) \text{ and } \ldots \text{ and } (\text{not } A_n))}$$

$$\mathbf{P(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - P(A_1' \cap A_2' \cap \ldots \cap A_n')}$$

For independent events

$$\mathbf{P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n) = 1 - P(\text{not } A_1) \bullet P(\text{not } A_2) \bullet \ldots \bullet P(\text{not } A_n)}$$

$$\mathbf{P(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - P(A_1') \bullet P(A_2') \bullet \ldots \bullet P(A_n')}$$

Example 4:

Often On time Parcel Service (*OOPS*) delivers a package to the wrong address with probability 0.05 on any delivery. Suppose that each delivery is independent of all the others. There were 7 packages delivered on a particular day. What is the probability that at least one of them was delivered to the wrong address?

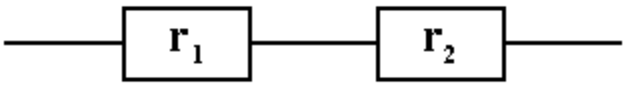
Example 5:

A major oil company has decided to drill independent test wells in the Alaskan wilderness. The probability of any well producing oil is 0.30. Find the probability that the fifth well is the first to produce oil.

Example 6:

An automobile salesman thinks that the probability of making a sale is 0.30. If he talks to five customers on a particular day, what is the probability that he will make exactly 2 sales? (Assume independence.)

Series Connection:

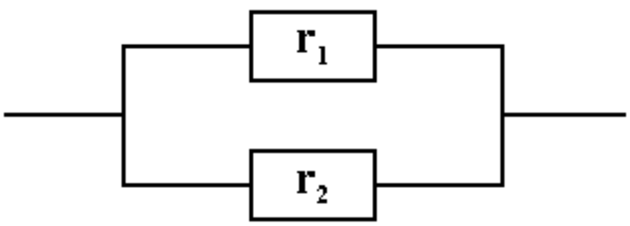


Reliability:

$$\mathbf{r}_1 \times \mathbf{r}_2$$

In general, $\mathbf{r}_1 \times \mathbf{r}_2 \times \ldots \times \mathbf{r}_k$

Parallel Connection:



Reliability:

$$1 - (1 - \mathbf{r}_1) \times (1 - \mathbf{r}_2)$$

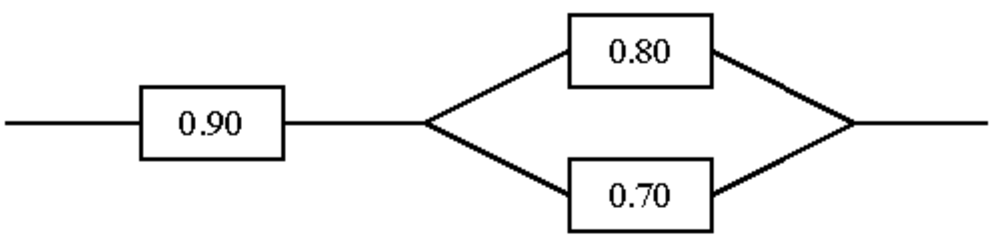
$$\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_1 \times \mathbf{r}_2$$

In general, $1 - (1 - \mathbf{r}_1) \times (1 - \mathbf{r}_2) \times \ldots \times (1 - \mathbf{r}_k)$

Example 7:

Compute the reliability of the following system of independent components

(the numbers represent the reliability of each component):



<u>Example 8</u>:
Alex and David agreed to play a series of tennis games (as many as needed) until one of them wins two games in a row. Alex will serve in the first game, then the serve would alternate game by game between Alex and David. David is a better tennis player; Alex has a 50% chance of winning a game on his serve and only a 20% chance of winning a game if David serves. Assume that all games are independent.

Find the probability that Alex is the first one to win two games in a row.