STAT 400
UIUC

1.5 – Bayes' Theorem

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<u>Review</u>:

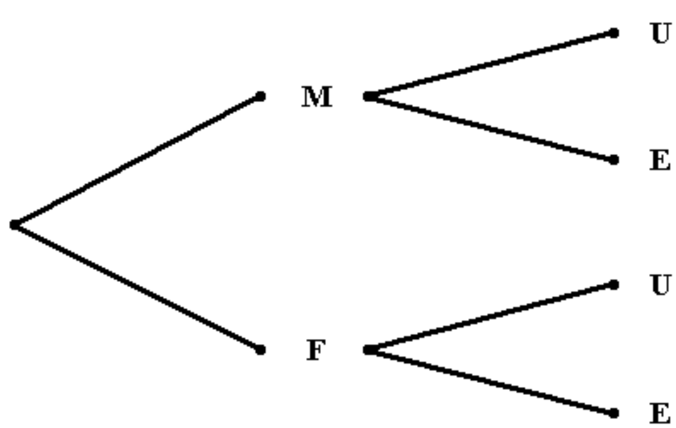
Law of Total Probability. $P(A) = P(A \cap B) + P(A \cap B')$

<u>Example 1</u>:
In Neverland, men constitute 60% of the labor force. The rates of unemployment are 6.0% and 4.5% among males and females, respectively. A person is selected at random from Neverland's labor force.

a) What is the probability that the person selected is a male <u>and</u> is unemployed?

b) What is the probability that the person selected is a female <u>and</u> is unemployed?

	Unemployed	**Employed**	Total
Male			
Female			
Total			



c) What is the probability that the person selected is unemployed?

Law of Total Probability.

$$P(A) = P(A \cap B) + P(A \cap B')$$
$$= P(B) \cdot P(A \mid B) + P(B') \cdot P(A \mid B')$$

In general,

$$P(A) = \sum_{i=1}^{m} P(B_i) \cdot P(A \mid B_i)$$

d) <u>Suppose the person selected is unemployed</u>. What is the probability that a male was selected?

Bayes' Theorem.

$$P(B \mid A) = \frac{P(B) \cdot P(A \mid B)}{P(B) \cdot P(A \mid B) + P(B') \cdot P(A \mid B')},$$

In general,

$$P(B_k \mid A) = \frac{P(B_k) \cdot P(A \mid B_k)}{\sum_{i=1}^{m} P(B_i) \cdot P(A \mid B_i)}, \quad k = 1, \ldots, m.$$

Example 2:

In a presidential race in Neverland, the incumbent Democrat (D) is running against a field of four Republicans (R_1, R_2, R_3, R_4) seeking the nomination. Political pundits estimate that the probabilities of R_1, R_2, R_3, and R_4 winning the nomination are 0.40, 0.30, 0.20, and 0.10, respectively. Furthermore, results from a variety of polls are suggesting that D would have a 55% chance of defeating R_1 in the general election, a 70% chance of defeating R_2, a 60% chance of defeating R_3, and an 80% chance of defeating R_4. Assuming all these estimates to be accurate, what are the chances that D will be a two-term president?

Example 3:

In Anytown, 10% of the people leave their keys in the ignition of their cars. Anytown's police records indicate that 4.2% of the cars with keys left in the ignition are stolen. On the other hand, only 0.2% of the cars without keys left in the ignition are stolen. Suppose a car in Anytown is stolen. What is the probability that the keys were left in the ignition?

<u>Example 4</u>:

In a certain population, the proportion of individuals who have a particular disease is 0.025. A test for the disease is positive in 94% of the people who have the disease and in 4% of the people who do not.

a) Find the probability of receiving a positive reaction from this test.

b) If a person received a positive reaction from this test, what is the probability that he/she has the disease?

c) If a person received a negative reaction from this test, what is the probability that he/she doesn't have the disease?