

**2.4 - Binomial Distribution**  
**2.5 - Geometric and Negative Binomial Dist.**

**Binomial Distribution:**

1. The number of trials,  $n$ , is fixed.
2. Each trial has two possible outcomes: “success” and “failure”.
3. The probability of “success”,  $p$ , is the same from trial to trial.
4. The trials are independent.
5.  $X$  = number of "successes" in  $n$  independent trials.

Then

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} = {}_n C_k \cdot p^k \cdot (1-p)^{n-k},$$

where  $k = 0, 1, \dots, n$ .

$$E(X) = n \cdot p \quad \text{Var}(X) = n \cdot p \cdot (1-p) \quad \text{SD}(X) = \sqrt{n \cdot p \cdot (1-p)}$$

**Example 1:**

Bart Simpson takes a multiple-choice exam in his Statistics 101 class. The exam has 15 questions, each has 5 possible answers, only one of which is correct. Bart did not study for the exam, so he guesses independently on every question.

Let  $X$  denote the number of questions that Bart gets right.

a) Is it appropriate to use Binomial model for this problem?

b) What is the expected number of questions that Bart would get right?

c) What is the probability that Bart answers exactly 3 questions correctly?

d) What is the probability that Bart would get at most 5 of the questions right?

e) What is the probability that Bart would get more than half of the questions right (i.e. what is the probability that Bart would get at least 8 of the questions right)?

f) Find the probability that Bart answers between 4 and 6 (including both 4 and 6) questions correctly?

| Binomial   | Outcome | Probability | k  | CDF at k   |
|------------|---------|-------------|----|------------|
| $n = 15$   | 0       | 0.03518437  | 0  | 0.03518437 |
| $p = 0.20$ | 1       | 0.13194140  | 1  | 0.16712577 |
|            | 2       | 0.23089744  | 2  | 0.39802321 |
|            | 3       | 0.25013890  | 3  | 0.64816210 |
|            | 4       | 0.18760417  | 4  | 0.83576628 |
|            | 5       | 0.10318229  | 5  | 0.93894857 |
|            | 6       | 0.04299262  | 6  | 0.98194119 |
|            | 7       | 0.01381906  | 7  | 0.99576025 |
|            | 8       | 0.00345476  | 8  | 0.99921501 |
|            | 9       | 0.00067176  | 9  | 0.99988677 |
|            | 10      | 0.00010076  | 10 | 0.99998754 |
|            | 11      | 0.00001145  | 11 | 0.99999899 |
|            | 12      | 0.00000095  | 12 | 0.99999994 |
|            | 13      | 0.00000006  | 13 | 1.00000000 |
|            | 14      | 0.00000000  | 14 | 1.00000000 |
|            | 15      | 0.00000000  |    |            |

**R:** `dbinom(x, size, prob)` `pbinom(q, size, prob)`

**Example 2:** ☺

An automobile salesman thinks that the probability of making a sale is 0.30. If he talks to five customers on a particular day, what is the probability that he will make exactly 2 sales? (Assume independence.)

**Example 3:** ☺

A major oil company has decided to drill independent test wells in the Alaskan wilderness. The probability of any well producing oil is 0.30. Find the probability that the fifth well is the first to produce oil.

**Geometric Distribution:**

$X$  = the number of **independent** trials until the first “success”.

Then

$$P(X = x) = (1 - p)^{x-1} \cdot p, \quad x = 1, 2, 3, \dots$$

$$E(X) = \frac{1}{p}, \quad \text{Var}(X) = \frac{1-p}{p^2}.$$

**R:** `rgeom(q, prob)` `dgeom(x, prob)`

**Example 4:**

A slot machine at a casino randomly rewards 15% of the attempts. Assume that all attempts are independent.

a) What is the probability that your first reward occurs on your fourth trial?

b) What is the probability that your first reward occurs on your seventh trial?

c) What is the probability that you get three rewards in ten trials?

d) What is the probability that your third reward occurs on your tenth trial?

### Negative Binomial Distribution:

$X$  = the number of **independent** trials until the  $k^{\text{th}}$  “success”.

Then

$$P(X = x) = \binom{x-1}{k-1} \cdot p^k \cdot (1-p)^{x-k}, \quad x = k, k+1, k+2, \dots$$

$$E(X) = \frac{k}{p} \quad V(X) = \frac{k \cdot (1-p)}{p^2}$$

**R:**            `pnbinom(q, size, prob)`                      `dnbinom(x, size, prob)`

### Example 4: (cont.)

e)     What is the probability that your fourth reward occurs on your fifteenth trial?

f)     What is the probability that you get four rewards in fifteen trials?