2.4 - Binomial Distribution (extended)

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Example 5:

Suppose that on Halloween 6 children come to a house to get treats. A bag contains 8 plain chocolate bars and 7 nut bars. Each child reaches into the bag and randomly selects 1 candy bar. Let X denote the number of nut bars selected.

a) Is the Binomial model appropriate for this problem?

b) Find the probability that exactly 2 nut bars were selected.

Hypergeometric Distribution:

N = population size,

S = number of "successes" in the population,

$N - S$ = number of "failures" in the population,

n = sample size.

X = number of "successes" in the sample when sampling is done without replacement.

Then

$$\mathrm{P}(X = x) = \frac{\binom{S}{x} \cdot \binom{N-S}{n-x}}{\binom{N}{n}} = \frac{{}_S C_x \cdot {}_{N-S} C_{n-x}}{{}_N C_n}$$

OR

$$\mathrm{P}(X = x) = \binom{n}{x} \cdot \left[\frac{S}{N} \cdot \frac{S-1}{N-1} \cdot \ldots \cdot \frac{S-x+1}{N-x+1} \right] \cdot \left[\frac{N-S}{N-x} \cdot \frac{N-S-1}{N-x-1} \cdot \ldots \cdot \frac{N-S-(n-x)+1}{N-n+1} \right]$$

$$\max(0, n + S - N) \le x \le \min(n, S).$$

R: `phyper(q, m, n, k)` `dhyper(x, m, n, k)`

Example 6:

A jar has N marbles, S of them are orange and $N - S$ are blue. Suppose 3 marbles are selected. Find the probability that there are 2 orange marbles in the sample, if the selection is done …

with replacement | without replacement

a) $N = 10,\ S = 4;$

b) $N = 100,\ S = 40;$

c) $N = 1{,}000,\ S = 400;$

	Binomial	**Hypergeometric**
	with replacement	without replacement
Probability	$P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$	$P(X = x) = \dfrac{\binom{S}{x} \cdot \binom{N-S}{n-x}}{\binom{N}{n}}$
Expected Value	$E(X) = n \cdot p$	$E(X) = n \cdot \dfrac{S}{N}$
Variance	$Var(X) = n \cdot p \cdot (1-p)$	$Var(X) = n \cdot \dfrac{S}{N} \cdot \left(1 - \dfrac{S}{N}\right) \cdot \dfrac{N-n}{N-1}$

If the population size is large (**compared to the sample size**) Binomial Distribution can be used regardless of whether sampling is with or without replacement.

Multinomial Distribution:

- The number of trials, n, is fixed.
- Each trial has k possible outcomes, with probabilities $p_1, p_2, \ldots, p_k$, respectively.

  $(p_1 + p_2 + \ldots + p_k = 1)$
- The trials are independent.
- $X_1, X_2, \ldots, X_k$ represent the number of times outcome 1, outcome 2, … , outcome k occur, respectively. $(X_1 + X_2 + \ldots + X_k = n)$

Then

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_k = x_k) = \frac{n!}{x_1!\,x_2!\ldots x_k!}\, p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k},$$

$$x_1 + x_2 + \ldots + x_k = n.$$

R: `dmultinom(x, prob)`

<u>Example 7</u>:

A particular brand of candy-coated chocolate comes in six different colors. Suppose 30% of all pieces are brown, 20% are blue, 15% are red, 15% are yellow, 10% are green, and 10% are orange. Thirty pieces are selected at random.

a) What is the probability that 10 are brown, 8 are blue, 7 are red, 3 are yellow, 2 are green, and none are orange?

b) What is the probability that 10 are brown, 8 are blue, and 12 are of other colors?

Example 8:

When Stéphane plays chess against his favorite computer program, he wins with probability 0.60, loses with probability 0.10, and 30% of the games result is a draw. Assume independence.

a) Find the probability that Stéphane's first win happens when he plays his third game.

b) Find the probability that Stéphane's fifth win happens when he plays his eighth game.

c) Find the probability that Stéphane wins 7 games, if he plays 10 games.

Now, assume Stéphane plays 12 games.

c) Find the probability that he wins 5 games, loses 3 games, and draws 4 games.

d) Find the probability that he wins 7 games, and draws 5 games.

e) Find the probability that Stéphane wins at least 8 games.

<u>Example 9</u>:

When correctly adjusted, a machine that makes widgets operates with a 5% defective rate. However, there is a 10% chance that a disgruntled employee kicks the machine, in which case the defective rate jumps up to 30%.

a) Suppose that a widget made by this machine is selected at random and is found to be defective. What is the probability that the machine had been kicked?

b) A random sample of 20 widgets was examined, 4 widgets out of these 20 are found to be defective. What is the probability that the machine had been kicked?

Hint:

- What is the probability of finding 4 defective widgets in a sample of 20, if (given) the machine has been kicked?
- What is the probability of finding 4 defective widgets in a sample of 20, if (given) the machine has not been kicked?