3.1 – Continuous Random Variables

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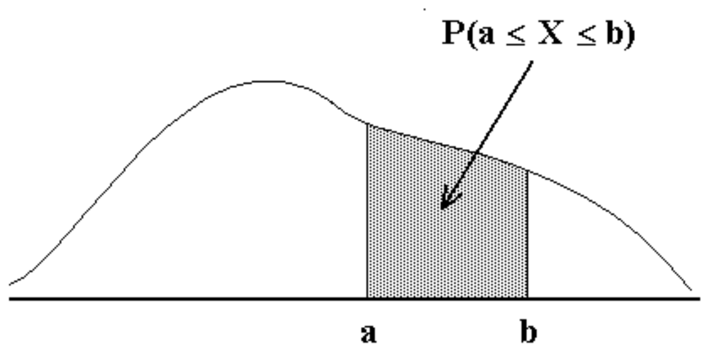
Continuous Random Variables

The probabilities associated with a continuous random variable X are determined by the **probability density function** of the random variable. The function, denoted $f(x)$, must satisfy the following properties:

1. $f(x) \geq 0$ for all x.
2. The total area under the entire curve of $f(x)$ is equal to 1.00.

Then the probability that X will be between two numbers a and b is equal to the area under $f(x)$ between a and b.



For any point c, $P(X = c) = 0.$

Therefore, $P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b).$

Expected value (mean, average): $\mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\, dx.$

Variance: $\sigma_X^2 = E\left[(X - \mu_X)^2\right] = \int_{-\infty}^{\infty} (x - \mu_X)^2 \cdot f(x)\, dx.$

$$\sigma_X^2 = E\left(X^2\right) - [E(X)]^2 = \left[\int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx\right] - (\mu_X)^2.$$

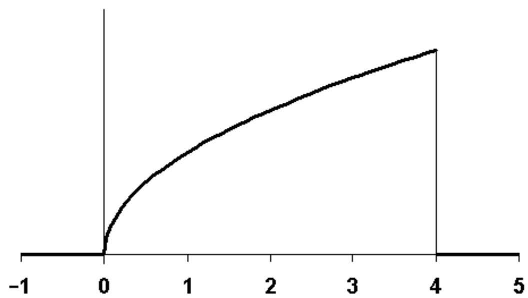
Moment Generating Function: $M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x)\, dx.$

Example 1:

Let X be a continuous random variable with the probability density function

$$f(x) = k \cdot \sqrt{x}, \quad 0 \le x \le 4,$$

$$f(x) = 0, \qquad \text{otherwise.}$$



a) What must the value of k be so that $f(x)$ is a probability density function?

b) Find the cumulative distribution function of X, $F_X(x) = P(X \le x)$.

c) Find the probability $P(1 \le X \le 2)$.

d) Find the median of the distribution of X. That is, find m such that

$$P(X \leq m) = P(X \geq m) = 1/2.$$

e) Find the 30th percentile of the distribution of X. That is, find a such that

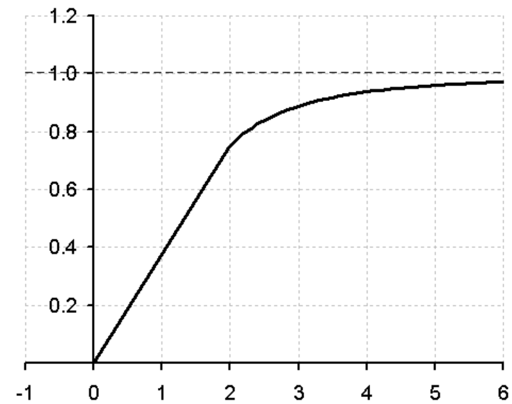
$$P(X \leq a) = 0.30.$$

f) Find $\mu_X = E(X)$.

g) Find $\sigma_X = SD(X)$.

<u>Example 2</u>:

Let X be a continuous random variable with the cumulative distribution function



$$F(x) = 0, \qquad x < 0,$$

$$F(x) = \frac{3}{8} \cdot x, \qquad 0 \le x \le 2,$$

$$F(x) = 1 - \frac{1}{x^2}, \qquad x > 2.$$

a) Find the probability density function $f(x)$.

b) Find the probability $P(1 \le X \le 4)$.

c) Find $\mu_X = E(X)$.

d) Find $\sigma_X = SD(X)$.