

Gamma Distribution:

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta},$$

$$0 \leq x < \infty$$

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x},$$

$$0 \leq x < \infty$$

$$E(X) = \alpha \theta$$

$$E(X) = \alpha / \lambda,$$

$$\text{Var}(X) = \alpha \theta^2$$

$$\text{Var}(X) = \alpha / \lambda^2$$

If T_α has a Gamma($\alpha, \theta = 1/\lambda$) distribution, where α is an integer, then

$$F_{T_\alpha}(t) = P(T_\alpha \leq t) = P(X_t \geq \alpha), \quad P(T_\alpha > t) = P(X_t \leq \alpha - 1),$$

where X_t has a Poisson($\lambda t = \frac{t}{\theta}$) distribution.

Example 1:

Alex is told that he needs to take bus #5 to the train station. He misunderstands the directions and decides to wait for the fifth bus. Suppose that the buses arrive to the bus stop according to Poisson process with the average rate of one bus per 20 minutes.

a) Find the probability that Alex would have to wait longer than 1 hour for the fifth bus to arrive.

b) Find the probability that the fifth bus arrives during the second hour.

Example 2:

Mistakes that David makes in class occur according to Poisson process with the average rate of one mistake per 10 minutes. Find the probability that the third mistake David makes occurs during the last 15 minutes of a 50-minute class.

$$\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du, \quad x > 0$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(x) = (x-1)\Gamma(x-1)$$

$$\Gamma(n) = (n-1)! \quad \text{if } n \text{ is an integer}$$