STAT 400
UIUC

3.3 – Normal Distribution

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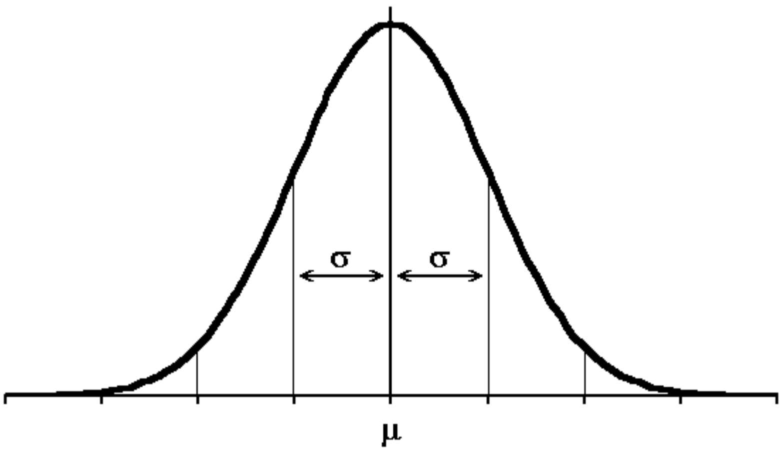
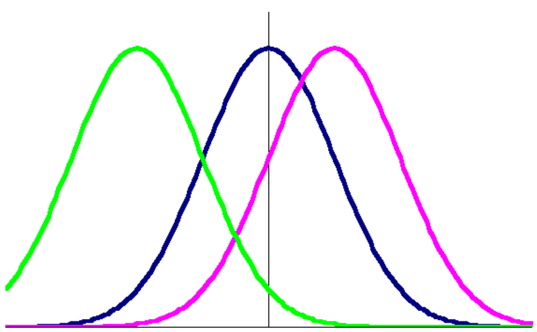


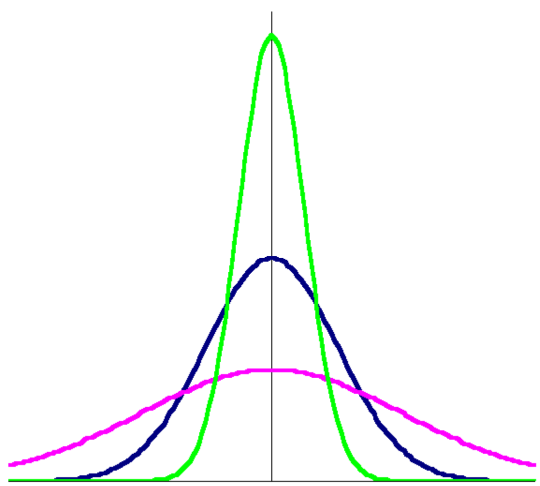
μ – mean

σ – standard deviation

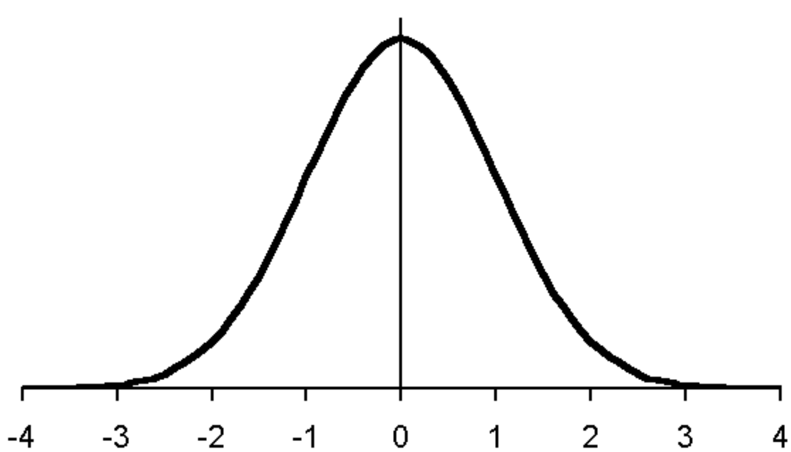
$$\mathbf{N}\left(\mu, \sigma^2\right)$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty.$$





Standard Normal Distribution.



mean

0

standard deviation

1

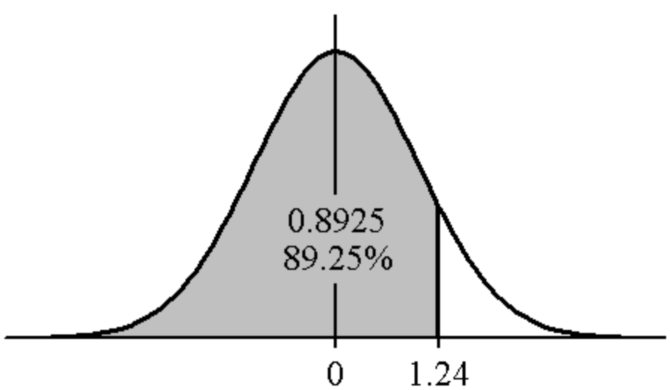
$$\mathrm{N}(0, 1)$$

Example:

For the standard normal distribution, find the area to the left of $z = 1.24$

1.24

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.0
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.53
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.57
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.61
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.65
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.68
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.72
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.75
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.78
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.81
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.83
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.86
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.88
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.90
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.91
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.93
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.94
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.95



Area to the left of $z = 1.24$ is **0.8925**.

$$Z \sim N(0,1)$$

$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X-\mu}{\sigma}$$

$$X = \mu + \sigma Z$$

<u>Example 1</u>:

At *Initech* , the salaries of the employees are normally distributed with mean $\mu = \$36,000$ and standard deviation $\sigma = \$5,000$.

a) Mr. Smith is paid \$42,000. What proportion of the employees of *Initech* are paid less that Mr. Smith?

b) What proportion of the employees have their salaries over \$40,000?

c) Suppose 10 *Initech* employees are randomly and independently selected. What is the probability that 3 of them have their salaries over \$40,000?

d) What proportion of the employees have their salaries between \$30,000 and \$40,000?

e) Mrs. Jones claims that her salary is high enough to just put her among the highest paid 15% of all employees working at *Initech* . Find her salary.

f) Ms. Green claims that her salary is so low that 90% of the employees make more than she does. Find her salary.

Example 2:

Suppose that the lifetime of ***Outlast*** batteries is normally distributed with mean $\mu = 240$ hours and unknown standard deviation. Suppose also that 20% of the batteries last less than 219 hours. Find the standard deviation of the distribution of the lifetimes.

Let X be normally distributed with mean μ and standard deviation σ.
Then the **moment-generating function** of X is

$$M_X(t) = e^{\mu t + \sigma^2 t^2 / 2}.$$

Let $Y = a X + b$. Then $M_Y(t) = e^{bt} M_X(at)$.

Therefore, Y is normally distributed with mean $a\mu + b$ and variance $a^2 \sigma^2$ (standard deviation $|a| \sigma$).

<u>Example 1</u>: (cont.)

g) All *Initech* employees receive a memo instructing them to put away 4% of their salaries plus $100 per month ($1,200 per year) in a special savings account to supplement Social Security. What proportion of the employees would put away more than $3,000 per year?