

4.1 – Joint Probability Distributions

Let X and Y be two discrete random variables. The **joint probability mass function** $p(x, y)$ is defined for each pair of numbers (x, y) by

$$p(x, y) = P(X = x \text{ and } Y = y).$$

Let A be any set consisting of pairs of (x, y) values. Then

$$P((X, Y) \in A) = \sum_{(x,y) \in A} p(x, y).$$

Let X and Y be two continuous random variables. Then $f(x, y)$ is the **joint probability density function** for X and Y if for any two-dimensional set A

$$P((X, Y) \in A) = \iint_A f(x, y) dx dy.$$

Example 1:

Consider the following joint probability distribution $p(x, y)$ of two random variables X and Y :

$x \setminus y$	0	1	2
1	0.15	0.10	0
2	0.25	0.30	0.20

a) Find $P(X + Y = 2)$.

b) Find $P(X > Y)$.

The **marginal probability mass functions** of X and of Y are given by

$$p_X(x) = \sum_{\text{all } y} p(x, y), \quad p_Y(y) = \sum_{\text{all } x} p(x, y).$$

The **marginal probability density functions** of X and of Y are given by

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx.$$

Example 1: (cont.)

c) Find the (marginal) probability distribution $p_X(x)$ of X and $p_Y(y)$ of Y.

If $p(x, y)$ is the joint probability mass function of (X, Y) OR $f(x, y)$ is the joint probability density function of (X, Y), then

discrete	continuous
$E(g(X, Y)) = \sum_{\text{all } x} \sum_{\text{all } y} g(x, y) \cdot p(x, y)$	$E(g(X, Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y) dx dy$

Example 1: (cont.)

d) Find $E(X)$, $E(Y)$, $E(X + Y)$, $E(X \cdot Y)$.

Example 2:

Alexis Nuts, Inc. markets cans of deluxe mixed nuts containing almonds, cashews, and peanuts. Suppose the net weight of each can is exactly 1 lb, but the weight contribution of each type of nut is random. Because the three weights sum to 1, a joint probability model for any two gives all necessary information about the weight of the third type.

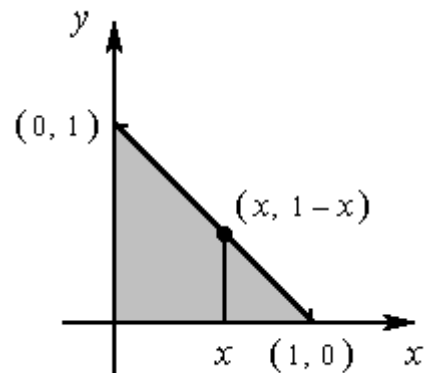
Let X = the weight of almonds in a selected can
and Y = the weight of cashews.

Then the region of positive density is $D = \{ (x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1 \}$.

Let the joint probability density function for (X, Y) be

$$f(x, y) = \begin{cases} 60x^2y & 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Verify that $f(x, y)$ is a legitimate probability density function.



b) Find the probability that the two types of nuts together make up less than 50% of the can. That is, find the probability $P(X + Y < 0.50)$. (Find the probability that peanuts make up over 50% of the can.)

c) Find the probability that there are more almonds than cashews in a can. That is, find the probability $P(X > Y)$.

d) Find the probability that there are at least twice as many cashews as there are almonds. That is, find the probability $P(2X \leq Y)$.

e) Find the marginal probability density function for X .

f) Find the marginal probability density function for Y .

g) Find $E(X)$, $E(Y)$, $E(X + Y)$, $E(X \cdot Y)$.

h) If 1 lb of almonds costs the company \$1.00, 1 lb of cashews costs \$1.50, and 1 lb of peanuts costs \$0.60, what is the expected total cost of the content of a can?