4.4 - Independent Random Variables

Stepanov
Dalpiaz
Nguyen

<u>Example 1</u>: (cont.)

Consider the following joint probability distribution $p(x, y)$ of two random variables X and Y:

x \ y	0	1	2	
1	0.15	0.10	0	0.25
2	0.25	0.30	0.20	0.75
	0.40	0.40	0.20	

Recall: A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

a) Are events $\{X = 1\}$ and $\{Y = 1\}$ independent?

Def Random variables X and Y are **independent** if and only if

discrete $p(x, y) = p_X(x) \cdot p_Y(y)$ for all x, y.

continuous $f(x, y) = f_X(x) \cdot f_Y(y)$ for all x, y.

$F(x, y) = P(X \le x, Y \le y)$.

$f(x, y) = \partial^2 F(x, y) / \partial x \, \partial y$.

Def Random variables X and Y are **independent** if and only if

$$F(x, y) = F_X(x) \cdot F_Y(y) \quad \text{for all } x, y.$$

b) Are random variables X and Y independent?

<u>Example 2</u>:

Let the joint probability density function for (X, Y) be

$$f(x,y)=\begin{cases} 60\,x^2\,y & 0 \le x \le 1,\ 0 \le y \le 1,\ x+y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Recall: $f_X(x) = 30\,x^2\,(1-x)^2, \quad 0 < x < 1,$

$f_Y(y) = 20\,y\,(1-y)^3, \quad 0 < y < 1.$

Are random variables X and Y independent?

<u>Example 3</u>:

Let the joint probability density function for (X, Y) be

$$f(x,y)=\begin{cases} x+y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

Example 4:

Let the joint probability density function for (X, Y) be

$$f(x,y)=\begin{cases} 12\,x\,(1-x)\,e^{-2y} & 0 \le x \le 1,\ y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

If random variables X and Y are independent, then

$$E\big(g(X)\cdot h(Y)\big) = E\big(g(X)\big)\cdot E\big(h(Y)\big).$$

Example 5:

Suppose the probability density functions of T_1 and T_2 are

$$f_{T_1}(x)=\alpha\, e^{-\alpha x}, \quad x>0, \qquad f_{T_2}(y)=\beta\, e^{-\beta y}, \quad y>0,$$

respectively. Suppose T_1 and T_2 are independent. Find $P(2T_1 > T_2)$.

Example 6:

Let X and Y be two independent random variables, X has a Geometric distribution with the probability of "success" $p = {}^1\!/_3$, Y has a Poisson distribution with mean 3. That is,

$$p_X(x) = \left(\frac{1}{3}\right) \cdot \left(\frac{2}{3}\right)^{x-1}, \qquad x = 1, 2, 3, \dots,$$

$$p_Y(y) = \frac{3^y e^{-3}}{y!}, \quad y = 0, 1, 2, 3, \dots .$$

a) Find $P(X = Y)$.

b) Find $P(X = 2Y)$.

a) Find $P(X > Y)$.