STAT 400
UIUC

5.8 - Chebyshev's Inequality

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Markov's Inequality:

Let $u(\mathrm{X})$ be a non-negative function of the random variable X.

If $\mathrm{E}[u(\mathrm{X})]$ exists, then, for every positive constant c,

$$\mathrm{P}(u(\mathrm{X}) \geq \mathrm{c}) \leq \frac{\mathrm{E}[u(\mathrm{X})]}{c}.$$

Chebyshev's Inequality:

Let X be any random variable with mean μ and variance σ^2. For any $\varepsilon > 0$,

$$\mathrm{P}(|\mathrm{X} - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$

or, equivalently,

$$\mathrm{P}(|\mathrm{X} - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}$$

Setting $\varepsilon = k\,\sigma$, $k > 1$, we obtain

$$\mathrm{P}(|\mathrm{X} - \mu| \geq k\,\sigma) \leq \frac{1}{k^2}$$

or, equivalently,

$$\mathrm{P}(|\mathrm{X} - \mu| < k\,\sigma) \geq 1 - \frac{1}{k^2}$$

That is, for any $k > 1$, the probability that the value of any random variable will be within k standard deviations of its mean is at least $1 - \frac{1}{k^2}$.

<u>Example 1</u>:

Suppose $\mu = E(X) = 17$, $\sigma = SD(X) = 5$. Find the lower bound of $P(9 < X < 25)$.

<u>Example 2</u>:

Suppose $\mu = E(X) = 17$, $\sigma = SD(X) = 5$.

Suppose also that the distribution of X is symmetric about the mean.
Find the lower bound of $P(10 < X < 30)$.