7.1 Confidence Intervals for Means
5.5 Student's t Distribution

Stepanov
Dalpiaz
Nguyen

Let $X_1, X_2, \ldots, X_n$ be i.i.d. $\mathbf{N}\left(\mu, \sigma^2\right)$.

Let

$$\overline{X} = \frac{\sum X_i}{n} = \frac{X_1 + X_2 + \ldots + X_n}{n} \qquad (\text{ sample mean })$$

$$S^2 = \frac{\sum\left(X_i - \overline{X}\right)^2}{n-1} \qquad (\text{ sample variance })$$

Then

$\overline{X}$ and S^2 are independent;

$\overline{X}$ has $\mathbf{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ distribution;

$\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$ has $\mathbf{N}(0,1)$ distribution;

$\dfrac{\sum\left(X_i - \mu\right)^2}{\sigma^2}$ has $\chi^2(n)$ distribution;

$\dfrac{(n-1)\cdot S^2}{\sigma^2} = \dfrac{\sum\left(X_i - \overline{X}\right)^2}{\sigma^2}$ has $\chi^2(n-1)$ distribution;

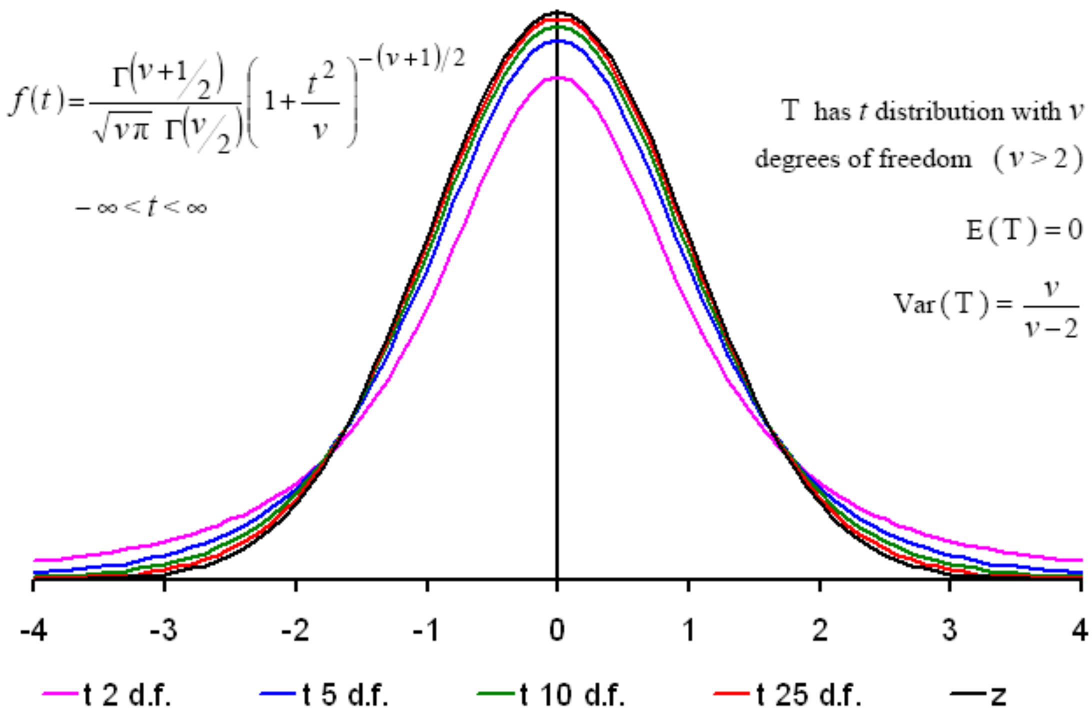
$\dfrac{\overline{X} - \mu}{S / \sqrt{n}}$ has $t(n-1)$ distribution.

A $(1-\alpha)\,100\%$ confidence interval for the population mean μ

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \qquad\qquad \overline{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$n-1$ degrees of freedom

The t Distribution



<u>Example 0</u>:

Let Z be a $N(0, 1)$ standard normal random variable.

Then $X = Z^2$ has a chi-square distribution with 1 degree of freedom.

<u>Example 1/2</u>:

Let X and Y be be two independent χ^2 random variables with m and n degrees of freedom, respectively.

Then $W = X + Y$ has a chi-square distribution with $m + n$ degrees of freedom.

Example 1:

A manufacturer of TV sets wants to find the average selling price of a particular model. A random sample of 25 different stores gives the mean selling price as $342 with a sample standard deviation of $14. Assume the prices are normally distributed. Construct a 95% confidence interval for the mean selling price of the TV model.

Example 2:

The following random sample was obtained from $N(\mu, \sigma^2)$ distribution:

16 12 18 13 21 15 8 17

a) Compute the sample mean and the sample standard deviation.

b) Construct a 95% confidence interval for μ.

c) Construct a 90% confidence upper bound for μ.

d) Construct a 99% confidence lower bound for μ.