Stepanov
Dalpiaz
Nguyen

7.3 - Confidence Intervals for Proportions

The **sample proportion**:

$$\hat{p} = \frac{x}{n}$$

where x is the number of elements in the sample found to belong to the category of interest (the number of "successes"), and n is the sample size.

$\mathrm{E}(\hat{P}) = p$, $\quad$ $\mathrm{Var}(\hat{P}) = \dfrac{p \cdot (1-p)}{n}$, $\quad$ $\mathrm{SD}(\hat{P}) = \sqrt{\dfrac{p \cdot (1-p)}{n}}$.

A large-sample confidence interval for the population proportion p is

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}.$$

<u>Example 1</u>:

Just prior to an important election, in a random sample of 749 voters, 397 preferred Candidate Y over Candidate Z. Construct a 90% confidence interval for the overall proportion of voters who prefer Candidate Y over Candidate Z.

	Upper-tail probability											
	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
z	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level											

Example 2:

An article on secretaries' salaries in the *Wall Street Journal* reports: "Three-fourth of surveyed secretaries said they make less than $25,000 a year." Suppose that the *Journal* based its results on a random sample of 460 secretaries drawn from every category of business. Give a 95% confidence interval for the proportion of secretaries earning less than $25,000 a year.

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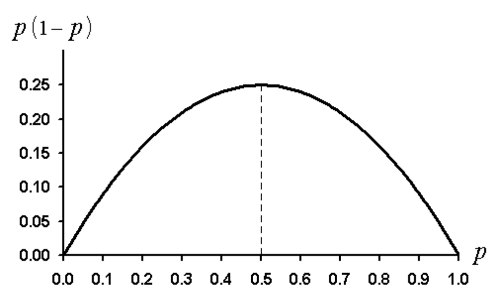
The sample size required to obtain a confidence interval for the population proportion p with specified margin of error ε is

$$n = \left(\frac{z_{\alpha/2}}{\varepsilon} \right)^2 p^* \left(1 - p^* \right).$$



Always round n up.

Conservative Approach: $p^* = 0.50$.

- If it is possible that $p = 0.50$, use $p^* = 0.50$.
- If it is not possible that $p = 0.50$, use p^* = the closest to 0.50 possible value of p.

Example 3:
Find the minimum sample size required for the overall proportion of voters who prefer Candidate Y over Candidate Z to within 2% with 90% confidence. (Assume that no guess as to what that proportion might be is available.)

Example 4:
A television station wants to estimate the proportion of the viewing audience in its area that watch its evening news. Find the minimum sample size required to estimate that proportion to within 3% with 95% confidence if …

a) no guess as to the value of that proportion is available.

b) it is known that the station's evening news reaches at most 30% of the viewing audience.