STAT 400
UIUC

Stepanov
Dalpiaz
Nguyen

8.1 - Hypothesis Tests for Variances

Null		Alternative	
$H_0: \sigma \ge \sigma_0$	vs.	$H_1: \sigma < \sigma_0$	
$H_0: \sigma^2 \ge \sigma_0^2$	vs.	$H_1: \sigma^2 < \sigma_0^2$	Left – tailed.
$H_0: \sigma \le \sigma_0$	vs.	$H_1: \sigma > \sigma_0$	
$H_0: \sigma^2 \le \sigma_0^2$	vs.	$H_1: \sigma^2 > \sigma_0^2$	Right – tailed.
$H_0: \sigma = \sigma_0$	vs.	$H_1: \sigma \ne \sigma_0$	
$H_0: \sigma^2 = \sigma_0^2$	vs.	$H_1: \sigma^2 \ne \sigma_0^2$	Two – tailed.

Test Statistic:

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma_0^2}$$

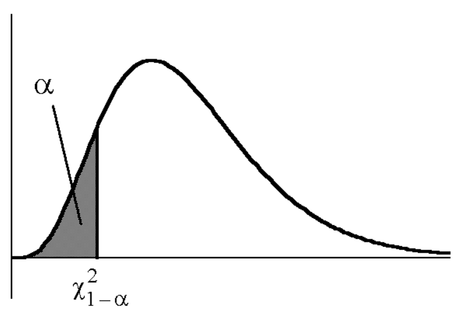
$n - 1$ degrees of freedom

Rejection Region:

$H_0: \sigma \ge \sigma_0$ vs. $H_1: \sigma < \sigma_0$

$H_0: \sigma^2 \ge \sigma_0^2$ vs. $H_1: \sigma^2 < \sigma_0^2$

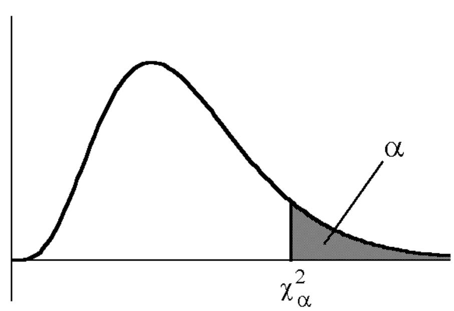
Left – tailed.



$H_0: \sigma \le \sigma_0$ vs. $H_1: \sigma > \sigma_0$

$H_0: \sigma^2 \le \sigma_0^2$ vs. $H_1: \sigma^2 > \sigma_0^2$

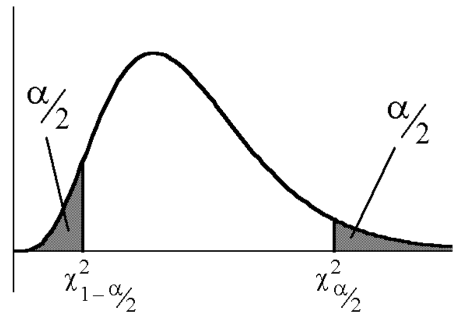
Right – tailed.



$H_0 : \sigma = \sigma_0$ vs. $H_1 : \sigma \neq \sigma_0$

$H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 \neq \sigma_0^2$

Two – tailed.



Example 1:

A machine at the Romano Drill Bit Company makes ½-inch ball bearings. When the machine is operating properly, the variance of the diameters of the bearings is at most 0.0003 inch^2. In a random sample of 41 bearings, the sample standard deviation of the diameters of the bearings was 0.02 inch. (Assume that the diameters of the bearings are approximately normally distributed.)

a) Perform the appropriate test at a 5% level of significance.

b) Perform the appropriate test at a 10% level of significance.

Example 2:

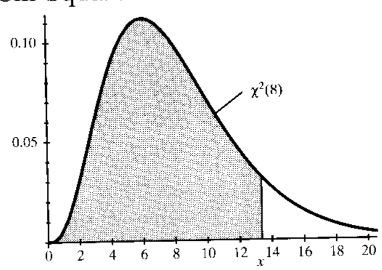
Container-filling machines are used to package a variety of liquids, including milk, soft drinks, and paint. Ideally, the amount of liquid should vary only slightly, since large variations will cause some containers to be underfilled (cheating the customer) and some to be overfilled (resulting in loss for the manufacturer). The president of a company that developed a new type of machine boasts that this machine can fill 1-litter (1,000-cm^3) containers so consistently that the standard deviation of the fills will be less than 0.5 cm^3. To examine the validity of the claim, a random sample of 25 1-litter fills was taken, and the sample standard deviation was 0.4 cm^3. Test the president's claim at the 5% significance level?

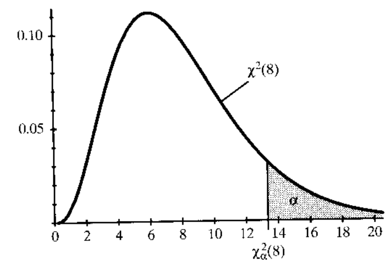
<u>Example 3</u>:

Metaltech Industries manufactures carbide drill tips used in drilling oil wells. The life of a carbide drill tip is measured by how many feet can be drilled before the tip wears out. Metaltech claims that under typical drilling conditions, the life of a carbide tip follows a normal distribution. Suppose that certain regulations require the variance of the lifetimes to be no more than 12 feet^2. Metaltech examines a random sample of 25 carbide tips to test whether these regulations are met.

Suppose Metaltech decided to use a 5% level of significance and the observed sample mean is 30.5 feet with the sample variance 16 feet^2. Perform the appropriate test.

Table IV The Chi-Square Distribution





$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

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