8.3 - Hypothesis Tests for One Proportion

Stepanov
Dalpiaz
Nguyen

<u>Example 1</u>:
A scientist wishes to test if a new treatment has a better cure rate than the traditional treatment which cures only 60% of the patients. In order to test whether the new treatment is more effective or not, a test group of 20 patients were given the new treatment. Assume that each personal result is independent of the others.

CDF @ x

n	x	p = **0.60**
20	0	0.000
	1	0.000
	2	0.000
	3	0.000
	4	0.000
	5	0.002
	6	0.006
	7	0.021
	8	0.057
	9	0.128
	10	0.245
	11	0.404
	12	0.584
	13	0.750
	14	0.874
	15	0.949
	16	0.984
	17	0.996
	18	0.999
	19	1.000

Trying to decide: cure rate $p \leq 0.60$ vs. $p > 0.60$.

a) If the new treatment has the same success rate as the traditional, what is the probability that at least 14 out of 20 patients (14 or more) will be cured?

b) Suppose that 14 out of 20 patients in the test group were cured. Based on the answer for part (a), is there a reason to believe that the new treatment has a better cure rate than the traditional treatment?

c) If the new treatment has the same success rate as the traditional, what is the probability that at least 17 out of 20 patients (17 or more) will be cured?

d) Suppose that 17 out of 20 patients in the test group were cured. Based on the answer for part (c), is there a reason to believe that the new treatment has a better cure rate than the traditional treatment?

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A **null hypothesis**, denoted by H_0, is an assertion about one or more population parameters. This is the assertion we hold as true until we have sufficient statistical evidence to conclude otherwise.

The **alternative hypothesis**, denoted by H_1, is the assertion of all situations *not* covered by the null hypothesis.

The test is designed to assess the strength of the evidence against the null hypothesis.

	H_0 true	**H_0 false**
Accept H_0 (**Do NOT Reject H_0**)	☺	Type II Error
Reject H_0	Type I Error	☺

α = significance level = P (Type I Error) = P (Reject H_0 | H_0 is true)

β = P (Type II Error) = P (Do Not Reject H_0 | H_0 is NOT true)

Power = 1 – P (Type II Error) = P (Reject H_0 | H_0 is NOT true)

Testing Hypotheses about a Population Proportion p

Null		Alternative	
$H_0: p \ge p_0$	vs.	$H_1: p < p_0$	Left – tailed.
$H_0: p \le p_0$	vs.	$H_1: p > p_0$	Right – tailed.
$H_0: p = p_0$	vs.	$H_1: p \ne p_0$	Two – tailed.

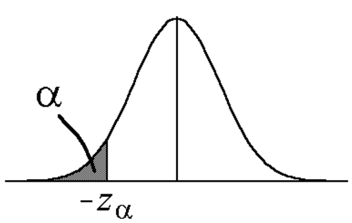
Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 \cdot (1 - p_0)}{n}}}. \qquad Z = \frac{Y - n \cdot p_0}{\sqrt{n \cdot p_0 \cdot (1 - p_0)}},$$

where Y is the number of S's in n independent trials.

Rejection Region:

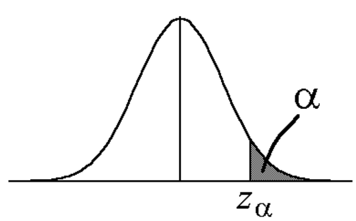
$H_0: p \ge p_0$
$H_1: p < p_0$
Left - tailed.



Reject H_0 if

$Z < -z_\alpha$

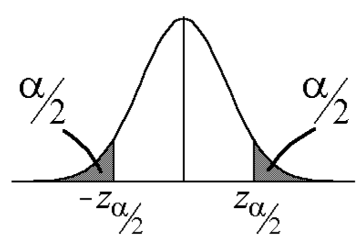
$H_0: p \le p_0$
$H_1: p > p_0$
Right - tailed.



Reject H_0 if

$Z > z_\alpha$

$H_0: p = p_0$
$H_1: p \ne p_0$
Two - tailed.



Reject H_0 if

$Z < -z_{\alpha/2}$

or

$Z > z_{\alpha/2}$

If the value of the Test Statistic falls into the Rejection Region,

then Reject H_0

otherwise, Accept H_0 (Do NOT Reject H_0)

Example 2:

A certain automobile manufacturer claims that at least 80% of its cars meet the tough new standards of the Environmental Protection Agency (EPA). Let p denote the proportion of the cars that meet the new EPA standards. The EPA tests a random sample of 400 its cars, suppose that 308 of the 400 cars in our sample meet the new EPA standards.

a) Perform an appropriate test at a 10% level of significance ($\alpha = 0.10$).

Claim:

H_0 : vs. H_1 :

Test Statistic:

Rejection Region:

Decision:

b) Perform an appropriate test at a 5% level of significance ($\alpha = 0.05$).

Rejection Region:

Decision:

The **P-value** (**observed level of significance**) is the probability, computed assuming that H_0 is true, of obtaining a value of the test statistic as extreme as, or more extreme than, the observed value.

(The smaller the p-value is, the stronger is evidence against H_0 provided by the data.)

P-value > α Do NOT Reject H_0 (Accept H_0).

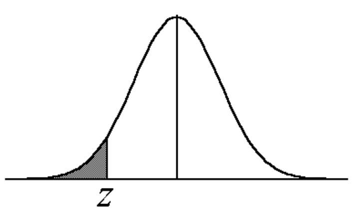
P-value < α Reject H_0.

Computing P-value:

$H_0: p \geq p_0$

$H_1: p < p_0$

Left - tailed.

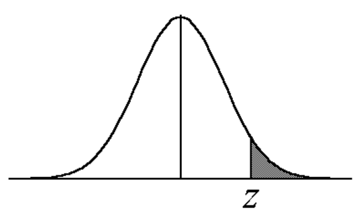


Area to the left of the observed test statistic

$H_0: p \leq p_0$

$H_1: p > p_0$

Right - tailed.

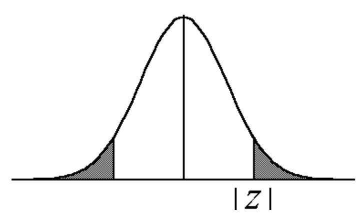


Area to the right of the observed test statistic

$H_0: p = p_0$

$H_1: p \neq p_0$

Two - tailed.



2 × area of the tail

c) Find the p-value of the appropriate test.

d) Using the p-value from part (c), state your decision (Accept H_0 or Reject H_0) at $\alpha = 0.08$.

Example 3:
Alex wants to test whether a coin is fair or not. Suppose he observes 477 heads in 900 tosses. Let p denote the probability of obtaining heads.

a) Perform the appropriate test using a 10% level of significance.

Claim:

H_0 : vs. H_1 :

Test Statistic:

Rejection Region:

Decision:

b) Find the p-value of the test in part (a).

c) Using the p-value from part (b), state your decision (Accept H_0 or Reject H_0) for $\alpha = 0.05$.